LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **APRIL 2025**

UMT 5501 – REAL ANALYSIS - II

Date: 24-04-2025 Dept. No. ______ Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

	SECTION A - K1 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
1.	**Fill in the blanks**
a)	The cluster point of the set $\left\{\frac{1}{n}\right\}, n = 1,2,3, \ldots$ is ______________.
b)	$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n =$ ______________.
c)	The value of 'c' in the mean value theorem for $f(x) = x(x-1)$ defined on $[1,2]$ is _____________.
d)	If every open cover of $A \subseteq \mathcal{R}$ has a finite subcover then A is ______________.
e)	A function which has only a finite number of distinct values, each being assigned on one or more subintervals of $[a, b]$ is called ______________.
2.	**MCQ**
a)	$\lim_{x\to\infty} x^n =$ a) 1 b) 0 c) ∞ d) none of these
b)	The function $f(x) = \|x-2\|$ at $x = 2$ is a) continuous and differentiable b) continuous but not differentiable c) differentiable but not continuous d) neither continuous nor differentiable
c)	The Rolle's constant 'c' for $f(x) = \|x\|$ in [-1,1] is a) 1 b) -1 c) 0 d) does not exist
d)	If $f: [a, b] \to \mathcal{R}$ is continuous then f is a) unbounded b) constant c) monotone d) Uniformly continuous
e)	Cantor set is a) open b) closed c) Both open and closed d) neither open nor closed
	SECTION A - K2 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
3.	**True or False**
a)	Not every polynomial function is continuous on $\mathcal{R}$.
b)	Every differentiable function on $\mathcal{R}$ is continuous of $\mathcal{R}$.
c)	$\lim_{x\to 0}\left(\frac{1}{x}\right)$ does not exist in $\mathcal{R}$.
d)	Unbounded function cannot be Riemann integrable.
e)	Union of infinitely many closed sets in $\mathcal{R}$ need not be closed.
4.	**Answer the following**
a)	Show that $\lim_{x\to 0}\left(x \sin\frac{1}{x}\right) = 0$.
b)	Prove that sine function is continuous on $\mathcal{R}$.
c)	Let $f: I \to \mathcal{R}$ has derivative at $c \in I$. Prove that f is continuous at c.
d)	Define Lipschitz function.
e)	Find the norm of the partition $P = \{0, 1.3, 2, 3.8, 5\}$ over $[0, 5]$.

SECTION B - K3 (CO2)	
Answer any TWO of the following in 100 words each.	**(2 x 10 = 20)**
5.	State and prove the sequential criterion for Limits.
6.	State and prove Bolzano's Intermediate value theorem.
7.	Show that $f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is differentiable at all points of $\mathcal{R}$ but does not have a continuous derivative f'.
8.	Prove that a function $f: \mathcal{R} \to \mathcal{R}$ is continuous if and only if $f^{-1}(G)$ is open in $\mathcal{R}$ whenever G is open in $\mathcal{R}$.
SECTION C – K4 (CO3)	
Answer any TWO of the following	**(2 x 10 = 20)**
9.	Prove that a number $c \in \mathcal{R}$ is a cluster point of a subset A of $\mathcal{R}$ if and only if there exist a sequence (a_n) in A such that $\lim(a_n) = c$ and $a_n \neq c$ for all $n \in \mathbb{N}$.
10.	Let $I = [a, b]$ be a closed bounded interval and $f: I \to \mathcal{R}$ be continuous on I. Prove that f has absolute maximum and absolute minimum on I.
11.	Deduce the proof of Taylor's theorem.
12.	If $f: [a, b] \to \mathcal{R}$ is continuous on $[a, b]$ then prove that $f \in \mathcal{R}[a, b]$.
SECTION D – K5 (CO4)	
Answer any ONE of the following	**(1 x 20 = 20)**
13.	a) State and prove Rolle's theorem. (10 marks) b) Let I be closed bounded interval and let $f: \to \mathcal{R}$ be continuous on I. Prove that f is uniformly continuous on I. (10 marks)
14.	a) State and prove Cauchy criterion for Riemann integrable functions. (10 marks) b) State and prove fundamental theorem of calculus. (10 marks)
SECTION E – K6 (CO5)	
Answer any ONE of the following	**(1 x 20 = 20)**
15.	Prove that a subset K of $\mathcal{R}$ is compact if and only if it is closed and bounded.
16.	a) State and prove the chain rule for differentiation. (10 marks) b) Let $f: [a, b] \to \mathcal{R}$ and let $c \in [a, b]$. Prove that $f \in \mathcal{R}[a, b]$ if and only if the restrictions of f to $[a, c]$ and $[c, b]$ are both Riemann integrable. Also prove that $\int_a^b f = \int_a^c f + \int_c^b f$. (10 marks)

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